

Normal State Properties of a Resonantly Interacting p -wave Fermi Gas

Juan Yao and Shizhong Zhang*

Department of Physics and Centre of Theoretical and Computational Physics,
The University of Hong Kong, Hong Kong, China

(Dated: October 6, 2016)

Motivated by the recent experimental progress in the study of p -wave resonant Fermi gases, we investigate the normal state properties of such a gas. We calculate the universal equation of state and the two p -wave contacts that characterise the universal properties of the system, in good agreement with experiments. Our calculation takes explicit account of the effective range correction, obtains the superfluid transition temperature T_c within the Nozières-Schmitt-Rink (NSR) scheme, and shows that it lies within experimental reach. We derive an analytic expression for T_c in the weak coupling limit and show explicitly the non-perturbative nature of the effective range corrections.

Introduction. Studies of p -wave Feshbach resonances with ultracold atomic gases date back to 2003 when they were first observed in ^{40}K [1]. This and subsequent experiments [2–11] have explored the collisional properties of the gas in detail and showed that the system suffers significant loss close to resonance. In contrast, broad s -wave Feshbach resonances are stabilized by Pauli exclusion of three-body processes, allowing an s -wave resonant superfluid to be realized [12]. The difference between the s - and p -wave resonances stems from the existence of centrifugal barrier in the case of p -wave resonance, which tends to support quasi-bound dimer states within the centrifugal barrier and thus leads to significant atom loss, preventing the study of a resonant p -wave gas in equilibrium [5, 13].

However, a recent experimental study [14] of a single component Fermi gas of ^{40}K , utilizing a fast spectroscopic measurement, has shown that close to the p -wave Feshbach resonance, the system can establish quasi-equilibrium between the scattering fermions and the quasi-bound dimer states, while suffering an overall loss that still allows the study of properties of the gases to be conducted. In this way, it is demonstrated experimentally that the p -wave resonant Fermi gas obeys a set of universal relations controlled by the p -wave contacts [15–19], analogous to the s -wave case [20–23]. Unlike the s -wave case where usually a single parameter, the s -wave scattering length, is sufficient for the description of the universal properties, in the p -wave case, one needs to take into account the effective range corrections in order to formulate a consistent theory [16]. In the presence of externally or spontaneous broken axial rotation symmetry, the p -wave contacts have to be extended to a tensor [24, 25] (see also Ref. [26]). So far, however, no explicit calculation of the p -contacts exists except via Virial expansion [16], leaving unexplored an exciting regime of p -wave resonant Fermi gases in the normal state, and in particular, close to the superfluid transition temperature.

In this Letter, we fill this gap by developing a many-body theory for the normal state of a single component p -wave Fermi gas. We adopt a two-channel formulation of p -wave resonances and extend the Nozières-Schmitt-

Rink (NSR) scheme for s -wave resonances [27], emphasizing the role of p -wave effective range. We calculate the universal equation of state for the resonant Fermi gases in the normal state and also the associated p -wave contacts. Finally we determine the superfluid transition temperature T_c of a p -wave Fermi gas, using parameters appropriate to the current experiment. An analytic expression for T_c is also obtained in the weak coupling limit that shows explicitly its non-perturbative dependence on the effective range, and further emphasizes its special importance as compared with s -wave case.

The Model. We adopt a two-channel description of the p -wave Feshbach resonance for a spinless Fermi gas. The non-interacting Hamiltonian:

$$\hat{H}_0 = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}} + \sum_{m, \mathbf{q}} (\epsilon_{\mathbf{q}}/2 - \nu_m) \hat{b}_{m, \mathbf{q}}^\dagger \hat{b}_{m, \mathbf{q}}, \quad (1)$$

where $\hat{a}_{\mathbf{k}}^\dagger$ is the creation operator for (spinless) fermions with momentum $\hbar\mathbf{k}$ with kinetic energy $\epsilon_{\mathbf{k}} = \hbar^2\mathbf{k}^2/2M$, where M is the fermion mass. $\hat{b}_{m, \mathbf{q}}^\dagger$ is the creation operator for closed channel molecules with momentum $\hbar\mathbf{q}$ and relative angular momentum projection m . In this work, we work close to a p -wave resonance and neglect other partial wave scatterings, including the s -wave. As a result, $m = 0, \pm 1$. In actual experiment for ^{40}K , the resonances for $m = 0$ and $m = \pm 1$ are split by about 0.5G [1, 14]. This is taken into account by assuming a m -dependent detuning ν_m of the closed channel molecules. The conversion between the open channel scattering fermions and the closed channel molecules is given by

$$\hat{V} = \sum_{m, \mathbf{k}, \mathbf{q}} \frac{g_m}{\sqrt{2V}} k Y_{1m}(\hat{\mathbf{k}}) \hat{a}_{\frac{\mathbf{q}}{2} - \mathbf{k}}^\dagger \hat{a}_{\frac{\mathbf{q}}{2} + \mathbf{k}}^\dagger \hat{b}_{m, \mathbf{q}} + \text{H.c.}, \quad (2)$$

where g_m is the coupling constant. The matrix element $k Y_{1m}(\hat{\mathbf{k}})$ arises from the p -wave symmetry of the molecules where $k = |\mathbf{k}|$ and $\hat{\mathbf{k}} = \mathbf{k}/k$. The bare coupling constants $\{g_m, \nu_m\}$ can be related to the low-energy p -wave scattering parameters by a standard renormalisa-

tion procedure (hereafter we set $\hbar = 1$) [14, 15]

$$v_m^{-1} = \frac{4\pi\nu_m}{g_m^2 M} + \frac{4\pi}{V} \sum_{\mathbf{k}} 1, \quad (3)$$

$$R_m^{-1} = \frac{4\pi}{g_m^2 M^2} + \frac{2\pi}{MV} \sum_{\mathbf{k}} \frac{1}{\epsilon_{\mathbf{k}}}, \quad (4)$$

where we have introduced the p -wave scattering volume v_m and effective range R_m through the low-energy effective range expansion $k^3 \cot \delta_m(k) = -1/v_m - k^2/R_m$. $\delta_m(k)$ is the p -wave scattering phase shift with magnetic projection m . Note that we have neglected the direct p -wave interaction between the spin polarized fermions. The total number of fermions N is given by $N = N_F + 2N_B \equiv N_F + 2 \sum_m N_{B,m}$, where $N_F = \sum_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}}$ and $N_{B,m} = \sum_{\mathbf{k}} b_{m,\mathbf{k}}^\dagger b_{m,\mathbf{k}}$. In the following, we define the Fermi wave vector k_F by $n \equiv N/V = k_F^3/(6\pi^2)$, where n is the number density and V is the volume of the system.

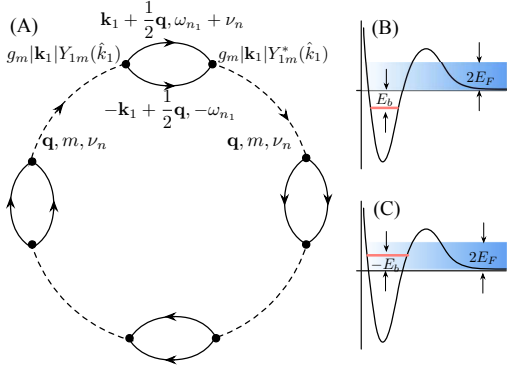


FIG. 1. (Color online) (A) Diagrams that contribute to the thermodynamic potential in the Nozières-Schmitt-Rink scheme. The solid lines represent the Green's functions for fermions $G_0^A(\mathbf{k}, i\omega_n) = (i\omega_n - (\epsilon_{\mathbf{k}} - \mu))^{-1}$. The dashed lines represent the Green's function for molecules $G_0^M(\mathbf{q}, i\nu_n)$ of Eq.(6). The vertex is given by $g_m|\mathbf{k}_1|Y_{1m}(\hat{k}_1)$, indicating the p -wave scattering through channel m . (B) Schematic diagrams of an actual bound state ($E_b > 0$) that is below the scattering threshold when $v_m > 0$. (C) For $v_m < 0$, there is a quasi-bound state in the continuum with energy $-E_b$ above the threshold. The scattering energy of two fermions extends to $2E_F$ in a degenerate Fermi gas.

Nozières-Schmitt-Rink scheme for p -wave gases. In the normal state, the thermodynamic potential Ω can be written as $\Omega = \Omega_0^F + \Omega_0^M + \Omega_{\text{int}}$, where $\Omega_0^F = -1/\beta \sum_{\mathbf{k}} \ln[1 + \exp(-\beta \xi_{\mathbf{k}})]$ gives the contribution of non-interacting fermions. $\beta = 1/k_B T$ and $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$ is the kinetic energy of fermions measured from its chemical potential μ . $\Omega_0^M = 1/\beta \sum_{m,\mathbf{q}} \ln(1 - \exp[-\beta(\epsilon_{\mathbf{q}}/2 - 2\mu - \nu_m)])$ gives the contribution from bosonic molecules. Note that while Ω_0^F depends only on physical parameters, the expression for Ω_0^M involves the bare detuning ν_m , which has to be renormalized later. Within NSR [27], the contribution to Ω from the inter-

action term is given by the ring diagrams in Fig. 1. Explicitly, we have

$$\Omega_{\text{int}} = \sum_{m,\mathbf{q}} \int_{-\infty}^{\infty} \frac{dz}{\pi} \frac{1}{e^{\beta z} - 1} \times \text{Im}\{\ln[1 + g_m^2 \Pi_m(\mathbf{q}, z + i0^+) G_0^M(\mathbf{q}, z + i0^+)]\}, \quad (5)$$

where $G_0^M(\mathbf{q}, i\nu_n)$ is the Green's function for non-interacting molecules,

$$G_0^M(\mathbf{q}, i\nu_n) = \frac{1}{i\nu_n - (\epsilon_{\mathbf{q}}/2 - \nu_m - 2\mu)} \quad (6)$$

and $i\nu_n = 2\pi n i/\beta \hbar$ is the bosonic Matsubara frequency with integer n . The polarization Π_m describes the propagation of two fermions and is given by

$$\Pi_m(\mathbf{q}, i\nu_n) = \frac{1}{V} \sum_{\mathbf{k}} \left\{ k^2 |Y_{1m}(\hat{\mathbf{k}})|^2 \times \frac{1 - f(\xi_{\mathbf{k}+\mathbf{q}/2}) - f(\xi_{-\mathbf{k}+\mathbf{q}/2})}{\xi_{\mathbf{k}+\mathbf{q}/2} + \xi_{-\mathbf{k}+\mathbf{q}/2} - i\nu_n} \right\}, \quad (7)$$

where $f(\xi) = (\exp(\beta\xi) + 1)^{-1}$ is the Fermi distribution function. A direct evaluation of Π_m shows that it is divergent and requires renormalization. This can be achieved together with the renormalization of Ω_0^M by noticing that

$$\ln[1 + g_m^2 \Pi_m G_0^M] = \ln g_m^2 + \ln G_0^M + \ln[g_m^{-2} (G_0^M)^{-1} + \Pi_m]. \quad (8)$$

The first term is a constant and can be neglected. The second term, when integrated in Eq. (5) cancels precisely Ω_0^M . One is thus left with the last term, which, by means of the renormalization conditions, Eqs. (3,4), reduces to

$$\Omega \equiv \Omega_0^F + \tilde{\Omega}_{\text{int}} = \Omega_0^F - \sum_{m,\mathbf{q}} \int \frac{dz}{\pi} \frac{1}{e^{\beta z} - 1} \delta_m(\mathbf{q}, z), \quad (9)$$

an expression of identical form as that for the single channel model [30]. Here $\delta_m(\mathbf{q}, z)$ is the p -wave scattering phase shift and is given by $\delta_m(\mathbf{q}, z) = -\text{Arg}[\Gamma_m^{-1}(\mathbf{q}, z)]$, where the vertex function is given by

$$\Gamma_m^{-1}(\mathbf{q}, z) = \frac{M^2}{4\pi R_m} \bar{z} + \frac{M}{4\pi v_m} + \Pi_m^r(\mathbf{q}, z). \quad (10)$$

The renormalized polarization $\Pi_m^r(\mathbf{q}, z)$ is

$$\Pi_m^r(\mathbf{q}, z) = \Pi_m(\mathbf{q}, z) - \frac{M}{V} \sum_{\mathbf{k}} 1 - \frac{M\bar{z}}{2V} \sum_{\mathbf{k}} \frac{1}{\epsilon_{\mathbf{k}}}, \quad (11)$$

where $\bar{z} = z - \epsilon_{\mathbf{q}}/2 + 2\mu$.

The structure of the vertex function Γ_m be analyzed most easily at high temperature where one can neglect the Fermi distribution factors in Eq. (7). In that limit we find analytically $\Gamma_m^{-1}(\mathbf{q}, z) = M^2/(4\pi R_m) \bar{z} + M/(4\pi v_m) + M^{5/2}/(4\pi)(-\bar{z})^{3/2}$. This leads to two bound states where $\bar{z} < 0$. Solving the equation $\Gamma_m^{-1}(\mathbf{q}, z) = 0$,

one finds $\bar{z}_1 = -R_m/(Mv_m)$ and $\bar{z}_2 = -1/(MR_m^2)$. We are mostly interested in the resonant regime where the dimensionless parameter $|k_F^3 v_m| \gg 1$, and correspondingly R_m assumes its natural scale in low-energy scattering. For all experiments studying p -wave resonances, $k_F R_m \ll 1$. This means that \bar{z}_2 corresponds to a very deeply bound state which lies outside the validity of effective range expansion used in our work. In fact, it is known that \bar{z}_2 corresponds to the *ghost field* where the normalization for such a state becomes negative [28, 29]. As a result, in the implementation of NSR calculation, we should neglect the \bar{z}_2 pole of Γ_m^{-1} , which is allowed if one is only interested in the low-energy properties of the system.

The other pole $\bar{z}_1 = -R_m/(Mv_m) < 0$ corresponds to a weakly bound state when $v_m > 0$. This will be referred to as the BEC side of the resonance [see Figure 1 (B)]. As one tunes across the resonance ($v_m = \pm\infty$) towards the BCS side ($v_m < 0$), the bound state emerges above zero energy and becomes a quasi-bound state because of the p -wave centrifugal barrier [see Figure 1 (C)]. The quasi-bound state couples strongly to scattering fermions and generates effective p -wave attraction between them. For $k_B T \ll |\bar{z}_2|$, it is then crucial to take into account the contribution from \bar{z}_1 pole, whose energy we denote as $E_{b,m} = R_m/(Mv_m)$ below.

In the high temperature limit $E_F \ll k_B T \ll |\bar{z}_2|$, $\tilde{\Omega}_{\text{int}}$ can be expanded to lowest order in fugacity $\exp(\beta\mu)$. We can write $\delta_m(\mathbf{q}, z) = \delta_m^M(\mathbf{q}, z) + \delta_m^F(\mathbf{q}, z)$ in Eq. (9), where $\delta_m^M(\mathbf{q}, z)$ arises from the molecular pole \bar{z}_1 and $\delta_m^F(\mathbf{q}, z)$ for the scattering fermions. In the absence of the many-body medium effects (neglecting the Fermi distribution function in Eq. (7)), one can reduce Eq. (9) to the standard virial expansion results where $\delta_m^M(\mathbf{q}, z)$ gives the contribution from bound molecules and $\delta_m^F(\mathbf{q}, z)$ for the scattering fermions [30]. At temperature $T > T_c$ and for a given set of scattering parameters $\{v_m, R_m\}$, one can solve for the thermodynamic potential in Eq. (9) together with the number equation $N = N_F + 2N_B = -\partial\Omega/\partial\mu$.

Universal equation of state. In experiment, the effective range R_m is approximately a constant around resonance and furthermore, independent of magnetic quantum number m . As a result, we set $R_m = R$ in the following for simplicity. On the other hand, the resonances for $m = 0$ (z -resonance) and $m = \pm 1$ (xy -resonance) are split due to magnetic dipole-dipole interactions [9]. In the vicinity of the xy -resonance, there are two possible molecular states in the closed channel, while for the z -resonance, there is only one. In ^{40}K experiments, these two resonances are well separated and do not interfere with each other which allows us to investigate the properties of the gas for the xy - and z -resonance separately.

The free energy of the system can be written in the following universal form around the xy - and z -resonances: $F_{xy,z} = NE_F f_{xy,z}(k_F R, -E_b/E_F, k_B T/E_F)$. In this

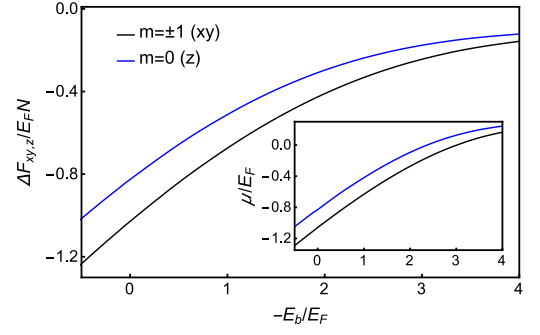


FIG. 2. Free energy $F_{xy,z}$ of the system as a function of $-E_b/E_F$ close to the xy - and z -resonances at $k_B T = 0.8E_F$. In our calculation, we have set $k_F R = 0.04$, appropriate to the experiment. F_{xy} (black line) is always smaller than that of the F_z (blue line) due to multiple molecular bound states. Inset shows the chemical potential μ as a function of $-E_b/E_F$ for the same set of parameters.

work, we focus on the dependences of $f_{xy,z}$ on E_b/E_F , assuming $k_F R = 0.04$, a typical experimental value. Note that the scaling form for $F_{xy,z}$ works for both normal and superfluid phases. For normal state, we calculate the scaling functions $f_{xy,z}$ within NSR for $k_B T = 0.8E_F$ (see Figure 2). For both resonances, the free energy decreases monotonically from the BCS side to the BEC side. Throughout the crossover, $f_{xy} < f_z$ since there exist two molecular bound states for xy -resonance and this lowers its free energy. In fact, the difference $|f_{xy} - f_z|$ increases as one moves towards the BEC limit. Close to resonance, the reduction of free energy is of order of Fermi energy, indicating strong p -wave interactions.

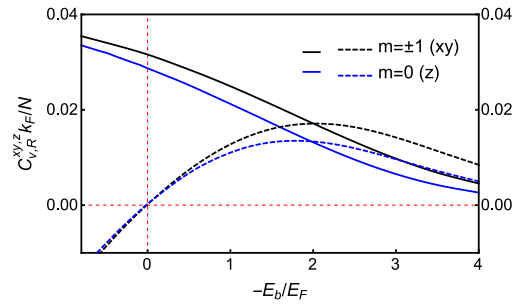


FIG. 3. Contacts $C_v^{xy,z}$ and $C_R^{xy,z}$ as a function of $-E_b/E_F$ for $k_B T = 0.8E_F$ and $k_F R = 0.04$. C_v^{xy} (solid black line) and C_v^z (solid blue line) decrease monotonically from the BEC to the BCS side with C_v^z always smaller than C_v^{xy} . On the other hand, both C_R^{xy} (dashed black line) and C_R^z (dashed blue line) vanish at resonance $v = \pm\infty$ and depend on $-E_b/E_F$ non-monotonically. The magnitude of C_R^z is always smaller than C_R^{xy} .

P-wave contacts. One of the most exciting aspects of the p -wave resonantly interacting Fermi gas is the existence of an extended set of universal relations involving

the p -wave contacts. As in the s -wave case, the p -wave contacts determine the universal properties of the system including their response to external radio-frequency field, which has been utilized to measure the p -wave contacts in recent experiment [14]. Theoretically, one can calculate the values of the relevant contacts using the adiabatic theorems, which relate them to the variation of the free energy with respect to the scattering parameters:

$$\left. \frac{\partial F}{\partial v_z^{-1}} \right|_{R,T} = -\frac{\hbar^2 C_v^z}{2M}; \quad \left. \frac{\partial F}{\partial R^{-1}} \right|_{v_z,T} = -\frac{\hbar^2 C_R^z}{2M}, \quad (12)$$

and the similar equation for the xy -resonance.

In Fig. 3, we show the calculated values of $C_v^{xy,z}$ and $C_R^{xy,z}$ within NSR for $k_F R = 0.04$ and $k_B T = 0.8 E_F$. $C_v^{xy,z}$ is monotonically decreasing as a function of $-E_b/E_F$ from the BCS to BEC side and is always positive, consistent with the general requirement [16]. C_v^{xy} is always greater than C_v^z , indicating a stronger dependences of F_{xy} on $-E_b/E_F$ than F_z due to multiple bound states in the closed channel. On the other hand, $C_R^{xy,z}$ shows non-monotonic behavior as a function of $-E_b/E_F$, with maximal value always achieved in the BCS side of the resonance. Note that even though $C_R^{xy,z}$ changes sign across the resonance and vanishes when $v = \pm\infty$, the magnitude of C_R^{xy} is always larger than C_R^z , consistent with stronger interactions around the xy -resonance.

Superfluid transition temperatures. Near a broad s -wave Feshbach resonance, superfluidity is the most robust at unitarity: the coherence length is shortest and the critical current is largest [12]. Near a p -wave resonance, the pairing symmetry is richer [31–35], and additional internal structure breaks scale invariance. In our formulation, apart from the splitting of resonance of $m = \pm 1$ (xy) and $m = 0$ (z), the vertex function Γ_m is diagonal in m . This means that close to xy -resonance around T_c , the superfluid order parameter is of the axial form, with gap function $\Delta_{\mathbf{k}}$ having the symmetry $Y_{1\pm 1}(\hat{\mathbf{k}})$ or their superposition. The detailed form cannot be obtained from our calculation within NSR. It is known that, however, the ground state order parameter should be of the pure $Y_{1\pm 1}(\hat{\mathbf{k}})$ form [31]. Close to the z -resonance, the order parameter is the standard polar form, for which $\Delta_{\mathbf{k}} \propto k_z$. Using the Thouless criterion, we can write the equation for T_c close to the z -resonance as $\Gamma_z^{-1}(\mathbf{0}, 0) = 0$, or explicitly

$$\frac{M^2 \mu}{2\pi R} + \frac{M}{4\pi v} + \Pi_z^r(\mathbf{0}, 0) = 0, \quad (13)$$

and similar equation determining the T_c for xy -resonance by replacing Π_z^r with Π_{xy}^r .

In Fig. 4, we show the calculated critical temperature for the xy - and z -resonance. For the xy -resonance, the critical temperature is always lower than that for the z -resonance. This is because for the xy -resonance, there are two molecular states in the closed channel which reduces

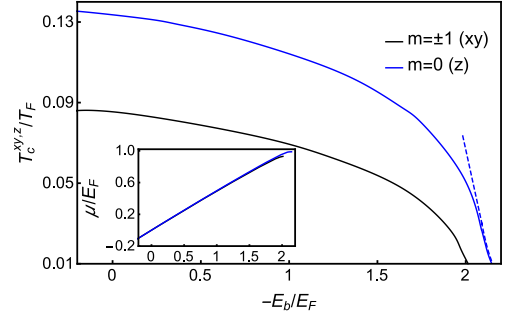


FIG. 4. Critical temperature T_c as a function of $-E_b/E_F$ for the xy (black line) and z -resonances (blue line). The dashed line is the asymptotic T_c in the BCS limit for the z -resonance given by Eq. (14). Inset shows the respective chemical potentials for the xy - and z -resonances.

the quantum degeneracy of the system. This is particularly evident in the BEC limit where all $N/2$ bosons are divided between two molecular states (with $m \pm 1$, each with number $N/4$ and density $n/4$), so the corresponding critical temperature for BEC is given by $T_c^{xy}/T_F = 0.086$. For the z -resonance, there is only one molecular state with $m = 0$ and the corresponding number density is $N/2V = n/2$. This gives a higher critical temperature $T_c^z/T_F = 0.137$. These values are also consistent with the calculated T_c via NSR in the BEC side. The general behavior of T_c in our calculation is consistent with that of Ref. [36], where, however, the dependence of T_c on the low-energy scattering parameters $\{v_m, R_m\}$ is implicit since it uses the bare coupling constants with a cutoff.

In the BCS limit, $v \rightarrow 0^-$, the attractive interaction is very weak and T_c becomes very small. In this limit, Eq. (13) simplifies considerably and one can obtain the asymptotic value of T_c as

$$T_c = \frac{8\gamma\tilde{\mu}}{\pi} T_F \exp\left[-\frac{8}{3}\tilde{\mu}^{\frac{3}{2}}\right] \exp\left[\frac{\pi\tilde{\mu}}{2k_F R} + \frac{\pi}{2k_F^3 v}\right], \quad (14)$$

where $\gamma = 1.78107$ is the exponential of the Euler constant and $\tilde{\mu} = \mu/E_F$. Note the standard exponential dependences on the p -wave interaction parameter, the scattering volume v , similar to the s -wave case. In addition, it should be noted that the dependence on the effective range R is also non-analytic, indicating the importance of finite range corrections in a dilute p -wave superfluid. Setting $\tilde{\mu} = 1$ in Eq. (14), one can equivalently write [37]

$$T_c = \frac{8\gamma}{\pi} T_F \exp\left[-\frac{8}{3}\right] \exp\left[\frac{\pi}{2k_F R} \left(1 - \frac{E_b}{2E_F}\right)\right], \quad (15)$$

showing that T_c changes rapidly around $E_b/E_F \approx 2$ when the quasi-bound p -wave state moves out of the scattering continuum.

At the critical temperature $T = T_c$, the corresponding chemical potential μ as a function of E_b/E_F is shown in

the inset of Fig. 4. The chemical potential μ is approximately half of the binding energy E_b and is thus linear in E_b over a wide range of interaction, but it quickly approaches E_F around $E_b/E_F \approx 2$ in the BCS limit. When $\mu = 0$, there is a very simple relation between the critical temperature and the scattering volume v

$$\left. \frac{1}{k_F^3 v} \right|_{\mu=0} = \frac{2 - \sqrt{2}}{2\sqrt{\pi}} \zeta(3/2) \left(\frac{T_c}{E_F} \right)^{3/2}, \quad (16)$$

where T_c is the critical temperature at $\mu = 0$. We note that this relation is *independent* of the effective range R .

Conclusion. In this Letter, we studied the normal state properties of a resonantly interacting p -wave Fermi gas. The universal equation of state, the p -wave contacts and the superfluid transition temperatures T_c are obtained using a two-channel formulation. We show that for the resonance with $m = \pm 1$, our estimation of the transition temperatures using actual experimental parameters is quite encouraging experimentally. There remain further important theoretical questions to be investigated such as the analogous Gor'kov-Melik-Barkhudarov correction to T_c from medium polarizations [38].

Acknowledgement. We thank Boyang Liu, Shanshan Ding and Joseph Thywissen for helpful discussions. This work is supported by Hong Kong Research Grants Council, General Research Fund, HKU 17306414, CRF, HKUST3/CRF/13G, and the Croucher Foundation under the Croucher Innovation Award.

* shizhong@hku.hk

- [1] C. A. Regal, C. Ticknor, J. L. Bohn, and D. S. Jin, *Phys. Rev. Lett.* **90**, 053201 (2003).
- [2] J. Zhang, E. G. M. van Kempen, T. Bourdel, L. Khaykovich, J. Cubizolles, F. Chevy, M. Teichmann, L. Tarruell, S. J. J. M. F. Kokkelmans, and C. Salomon, *Phys. Rev. A* **70**, 030702(R) (2004).
- [3] Kenneth Günter, Thilo Stöferle, Henning Moritz, Michael Köhl, and Tilman Esslinger, *Phys. Rev. Lett.* **95**, 230401 (2005).
- [4] C. H. Schunck, M. W. Zwierlein, C. A. Stan, S. M. F. Raupach, W. Ketterle, A. Simoni, E. Tiesinga, C. J. Williams, and P. S. Julienne, *Phys. Rev. A* **71**, 045601 (2005).
- [5] J. P. Gaebler, J. T. Stewart, J. L. Bohn, and D. S. Jin, *Phys. Rev. Lett.* **98**, 200403 (2007).
- [6] J. Fuchs, C. Ticknor, P. Dyke, G. Veeravalli, E. Kuhnle, W. Rowlands, P. Hannaford, and C. J. Vale, *Phys. Rev. A* **77**, 053616 (2008).
- [7] Y. Inada, M. Horikoshi, S. Nakajima, M. Kuwata-Gonokami, M. Ueda, and T. Mukaiyama, *Phys. Rev. Lett.* **101**, 100401 (2008).
- [8] Takuya Nakasuji, Jun Yoshida, and Takashi Mukaiyama, *Phys. Rev. A* **88**, 012710 (2013).
- [9] C. Ticknor, C. A. Regal, D. S. Jin, and J. L. Bohn, *Phys. Rev. A* **69**, 042712 (2004).
- [10] F. Chevy, E.G.M. van Kempen, T. Bourdel, J. Zhang, L. Khaykovich, M. Teichmann, L. Tarruell, S.J.J.M.F. Kokkelmans, C. Salomon, *Phys. Rev. A* **71**, 062710 (2005).
- [11] Muhammad Waseem, Zhiqi Zhang, Jun Yoshida, Keita Hattori, Taketo Saito, Takashi Mukaiyama, arXiv:1607.02270 (2016)
- [12] W. Zwerger, ed., *The BCS-BEC Crossover and the Unitary Fermi Gas*, (Springer-Verlag 2011).
- [13] J. Levinsen, N. R. Cooper, and V. Gurarie, *Phys. Rev. A* **78**, 063616 (2008).
- [14] Christopher Luciuk, Stefan Trotzky, Scott Smale, Zhenhua Yu, Shizhong Zhang, and Joseph H. Thywissen, *Nature Physics* **12**, 599-605 (2016).
- [15] Shuhei M. Yoshida and Masahito Ueda, *Phys. Rev. Lett.* **115**, 135303 (2015)
- [16] Zhenhua Yu, Joseph H. Thywissen, and Shizhong Zhang, *Phys. Rev. Lett.* **115**, 135304 (2015). Erratum, *ibid.* **117**, 019901 (2016)
- [17] Mingyuan He, Shaoliang Zhang, Hon Ming Chan, and Qi Zhou, *Phys. Rev. Lett.* **116**, 045301 (2016)
- [18] Shi-Guo Peng, Xia-Ji Liu, Hui Hu, arXiv:1607.03989 (2016)
- [19] Xiaoling Cui and Huifang Dong, arXiv:1608.00183 (2016)
- [20] Shina Tan, *Ann. Phys. N.Y.* **323**, 2952 (2008).
- [21] E. Braaten and L. Platter, *Phys. Rev. Lett.* **100**, 205301 (2008).
- [22] S. Zhang and A. J. Leggett, *Phys. Rev. A* **79**, 023601 (2009).
- [23] F. Werner, L. Tarruell, Y. Castin, *Euro. Phys. J. B* **68**, 410 (2009).
- [24] Shao-Liang Zhang, Mingyuan He and Qi Zhou, arXiv:1606.05176 (2016).
- [25] Shuhei M. Yoshida and Masahito Ueda, arXiv:1606.07235 (2016).
- [26] Ran Qi, arXiv:1606.03299v3 (2016).
- [27] P. Nozières, S. Schmitt-Rink, *Journal of Low Temperature Physics* **59**, 195 (1985).
- [28] Yusuke Nishida, *Phys. Rev. A* **86**, 012710 (2012).
- [29] Eric Braaten, P. Hagen, H.-W. Hammer, and L. Platter, *Phys. Rev. A* **86**, 012711 (2012).
- [30] C. J. Pethick and H. Smith, *Bose-Einstein Condensation in Dilute Gases*, 2nd edition, Cambridge University Press (2008).
- [31] T.-L. Ho and R. B. Diener, *Phys. Rev. Lett.* **94**, 090402 (2005).
- [32] C.-H. Cheng and S. K. Yip, *Phys. Rev. Lett.* **95**, 070404 (2005).
- [33] V. Gurarie, L. Radzihovsky, A.V. Andreev, *Phys. Rev. Lett.* **94**, 230403 (2005).
- [34] C.-H. Cheng and S.-K. Yip, *Phys Rev B* **73**, 064517 (2006).
- [35] V. Gurarie, L. Radzihovshy, *Annals of Physics* **332**, 2-119 (2007).
- [36] Y. Ohashi, *Phys. Rev. Lett.* **94**, 050403 (2005).
- [37] M. Iskin and C. A. R. Sa de Melo, *Phys. Rev. Lett.* **96**, 040402 (2006).
- [38] L. P. Gorkov and T. K. Melik-Barkhudarov, *Sov. Phys. JETP* **13**, 1018 (1961).